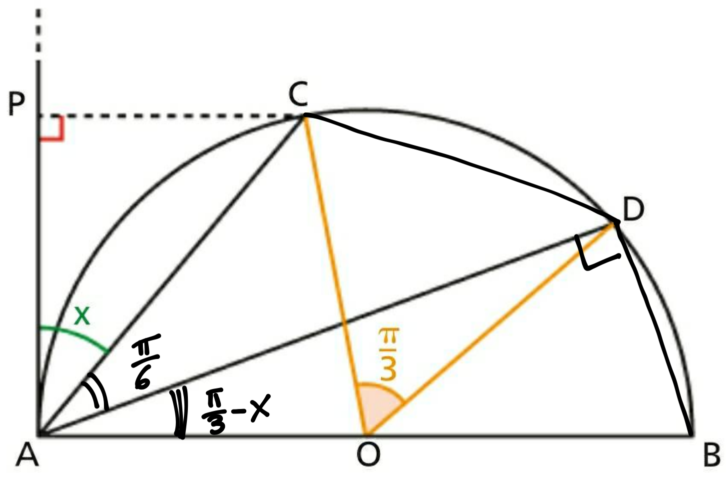
25/10/2019

585 Nella semicirconferenza di diametro $\overline{AB} = 2r$ in figura, esprimi in funzione dell'angolo x il rapporto tra $\overline{AP} \cdot \overline{CD}$ e l'area del triangolo ACD.



Calcola quindi il limite di tale rapporto al tendere di C ad A. [4]

$\overset{\triangle}{COD}$ è equilatero

$\overline{CD} = r$

$C\hat{A}D = \frac{1}{2}\frac{\pi}{3}$ perché angolo alla circonferenza corrispondente all'angolo al centro $C\hat{O}D$

$0 < x < \frac{\pi}{3}$ Da calcolare $\lim_{x\to 0^+} \frac{\overline{AP} \cdot \overline{CD}}{\mathcal{A}_{ACD}}$

$\overline{AP} = \overline{CA} \cdot \cos x$ $\mathcal{A}_{ACD} = \frac{1}{2}\, \overline{CA} \cdot \overline{AD} \cdot \sin\frac{\pi}{6}$

$\overline{AD} = 2r\cos\left(\frac{\pi}{3} - x\right)$

$$\frac{\overline{AP} \cdot \overline{CD}}{\mathcal{A}_{ACD}} = \frac{\cancel{r} \cdot \cancel{\overline{CA}} \cdot \cos x}{\cancel{\frac{1}{2}}\cancel{\overline{CA}} \cdot \cancel{2r}\cos\left(\frac{\pi}{3} - x\right) \cdot \frac{1}{2}} = \frac{2\cos x}{\cos\left(\frac{\pi}{3} - x\right)}$$

$$\lim_{x\to 0^+} \frac{2\cos x}{\cos\left(\frac{\pi}{3} - x\right)} = \frac{2 \cdot 1}{\cos\frac{\pi}{3}} = \frac{2}{\frac{1}{2}} = \boxed{4}$$

445 $\lim_{x \to +\infty} x[\ln(x^2+4) - 2\ln x] =$

$$= \lim_{x\to+\infty} x\left[\ln(x^2+4) - \ln x^2\right] =$$

$$= \lim_{x\to+\infty} x \ln\left(\frac{x^2+4}{x^2}\right) = \lim_{x\to+\infty} x \ln\left(1+\frac{4}{x^2}\right) =$$

$$= \lim_{x\to+\infty} \frac{\ln\left(1+\frac{4}{x^2}\right)}{\frac{1}{x}\cdot\frac{4x}{4x}} = \lim_{x\to+\infty} \frac{\overset{\nearrow 1}{\ln\left(1+\frac{4}{x^2}\right)}}{\frac{4}{x^2}\cdot\frac{x}{4}} = \frac{1}{+\infty} = 0$$

$$\frac{4}{x^2} = t$$

$$x \to +\infty \Rightarrow t \to 0^+$$

$$\lim_{t\to0^+} \frac{\overset{\nearrow 1}{\ln(1+t)}}{t\cdot\underbrace{\frac{1}{2\sqrt{t}}}_{\to +\infty}} = 0$$

$$x = \frac{2}{\sqrt{t}} \Rightarrow \frac{x}{4} = \frac{1}{2\sqrt{t}}$$

444 $\lim_{x\to\infty}\left(\frac{3x-1}{3x+2}\right)^{\frac{x}{2}} = 1^{\infty}$ F.I.

$$\lim_{x\to\infty} e^{\frac{x}{2}\ln\left(\frac{3x-1}{3x+2}\right)}$$

$$\frac{3x-1}{3x+2} = \frac{3x+2-2-1}{3x+2} = \frac{3x+2}{3x+2} - \frac{3}{3x+2} = 1 - \frac{3}{3x+2}$$

$$\frac{x}{2}\ln\left(1-\frac{3}{3x+2}\right) = \frac{\ln\left(1-\frac{3}{3x+2}\right)}{\frac{2}{x}} = (*)$$

pongo

$$t = -\frac{3}{3x+2} \Rightarrow 3x+2 = -\frac{3}{t} \Rightarrow 3x = -2-\frac{3}{t}$$

$$\Downarrow$$

$t \to 0$ per $x\to\infty$

$$x = -\frac{2}{3}-\frac{1}{t} = \frac{-2t-3}{3t}$$

$$(*) = \frac{\ln(1+t)}{\frac{6t}{-2t-3}} = \underbrace{\frac{\ln(1+t)}{6t}}_{\to 1}(\underbrace{-2t}_{\to 0}-3) \xrightarrow[t\to 0]{} \frac{1}{6}(-3) = -\frac{1}{2}$$

in definitiva

$$\lim_{x\to\infty} e^{\frac{x}{2}\ln\left(\frac{3x-1}{3x+2}\right)} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$